



VIBROACOUSTIC ANALYSIS OF A FLUID-LOADED CYLINDRICAL SHELL EXCITED BY A ROTATING LOAD

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A modal analysis study is presented to evaluate the vibroacoustic behaviour of a cylindrical thin elastic shell of finite length, extended by two semi-infinite perfectly reflecting cylinders. The shell is immersed in a stationary and non-viscous fluid extending up to infinity, and excited by a constant point load travelling continuously along the circumferential direction at a rotating speed Ω . An expression of modal displacement amplitude of the submerged shell is given, which shows that the shell is exclusively excited only when the circumferential mode order *n* equals the harmonic order *N* of the rotating load. For this rotating excitation, the vibration and radiation critical speeds are identified. Extensive numerical results are also presented to illustrate the comparison of vibration and sound radiation ability in two different fluids, which show that the behaviour of the shell in water is very different from that of the one in air.

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1. INTRODUCTION

A fluid-loaded cylindrical shell is the basic structural element widely used in many industrial fields. Sound radiation from this kind of shell is still an important subject in the area of noise control up to now. Whether a fluid is considered as light or heavy has been defined in reference [1], in which air can be considered as a light fluid and water as a heavy fluid in general. Extensive studies on the vibroacoustic characteristics of the shell subjected to non-moving mechanical point forces have been done by many researchers [1-8] in the past. However, there are a large number of cases of practical interest to the industry in which a fluid-loaded cylindrical shell is excited by a moving load travelling continuously along its circumference. Liao and Kessel [9], Bogy et al. [10], Shirakawa [11], and Huang and Soedel [12] have studied the dynamic response of a simply supported cylindrical shell excited by a circumferentially moving point load of constant amplitude. Furthermore, Panneton et al. [13] have studied the vibration and sound radiation behaviour based on previous studies. However, their research was limited only to the case of a light fluid (e.g. air), and the interaction between the shell and the fluid was neglected. For the case of a heavy fluid (e.g., water), there exists a strong structural-acoustic coupling between the shell and the heavy fluid, and the radiation impedance cannot be neglected. This kind of problem is more difficult to

analyze and involves a feedback mechanism (cf. reference [2]). Because of this interaction or coupling, to the author's knowledge, no work has thoroughly investigated the case of a heavy fluid-loaded shell under a rotating load. This is the aim of this paper.

To understand the vibroacoustic behaviour of this subject, we use numerical results based on the well-known modal analysis method frequently used in the area of structural-acoustic coupling research. Results for radiated power, shell quadratic velocity and radiation factor are presented and discussed for air and water. In particular, the phenomena related to the vibration and radiation critical speeds are analyzed for the two different fluids.

2. BASIC THEORY

The problem under consideration is schematically depicted in Figure 1: a thin elastic cylindrical shell (exterior radius *a*, length *L*, thickness *h*) simply supported along its two end circumference is immersed in an unbounded fluid whose density is ρ_0 and sound speed is c_0 . The fluid is assumed stationary and non-viscous. The shell is terminated by two semi-infinite cylindrical rigid baffles, and is excited at $z = z_0$ by a point load rotating with an angular velocity Ω along the circumference. A system of cylindrical co-ordinates (r, φ, z) is used to define position of points on the shell surface or in the exterior acoustic mediums. It is assumed that no other energy in the fluid and the interior of the shell is occupied by vacuo. Then the problem of vibroacoustic coupling only exists between the shell and the exterior fluid. This kind of problem can be solved by the vibration equation of the shell, the Helmholtz equation in the fluid and the corresponding boundary conditions. In subsequent studies, the time variation factor $e^{-j\omega t}$ is suppressed for the sake of brevity.

According to the classical assumption of Donnell, the shell equations of motion are well known as

$$\frac{Eh}{1-v^2} [\mathbf{L}_D] \mathbf{U} + \rho h \omega^2 [\mathbf{I}] \mathbf{U} = \mathbf{F},$$
(1)



Figure 1. Fluid-loaded cylindrical shell and co-ordinate system.

where *E*, *v*, and ρ are the Young's modulus, the Poisson ratio, and the density of the shell material respectively. $\mathbf{U} = \{u_z(a, \varphi, z), u_\varphi(a, \varphi, z), u_r(a, \varphi, z)\}^T$ is the vector of the shell displacements in the *z*, φ , *r* directions at point (a, φ, z) on the shell surface. **F** is the force vector per unit area exciting the shell:

$$\mathbf{F} = \begin{cases} 0\\ 0\\ -f(a,\varphi,z) + p(a,\varphi,z) \end{cases},\tag{2}$$

where $f(a, \varphi, z)$ is the rotating force given by reference [13], and $p(a, \varphi, z)$ is the surface acoustic pressure. Finally, $[\mathbf{L}_D]$ denotes the classical Donnell differential operator for thin shell theory (cf. reference [14]), and [I] is a unit matrix.

It is well known that the Helmholtz equation for the sound pressure $p(r, \varphi, z)$ in exterior acoustic fluid medium is

$$(\nabla^2 + k^2) p(r, \varphi, z) = 0,$$
(3)

where k is the sound wavenumber in fluids. The sound pressure $p(r, \varphi, z)$ also satisfies the following boundary condition:

$$\frac{\partial p}{\partial r}(r,\varphi,z)\Big|_{r=a} = \begin{cases} \omega^2 \rho_0 u_r(a,\varphi,z), & 0 \le x \le L, \\ 0 & \text{otherwise.} \end{cases}$$
(4)

To solve the sound pressure $p(r, \varphi, z)$ in equation (3), the Green function $G(r, \varphi, z | a, \varphi_1, z_1)$ and corresponding boundary condition for the exterior Neumann problem must be introduced. Thus, the representation of the shell surface sound pressure $p(a, \varphi, z)$ is written as

$$p(a, \varphi, z) = -\rho_0 \omega^2 \int_s G(a, \varphi, z | a, \varphi_1, z_1) u_r(a, \varphi_1, z_1) dS,$$
(5)

where $u_r(a, \varphi_1, z_1)$ is the radial displacement component at another point (a, φ_1, z_1) on the shell surface, and S is the surface area of the shell; $G(a, \varphi, z | a, \varphi_1, z_1)$ is given in reference [5] as follows:

$$G(a, \varphi, z | a, \varphi_1, z_1) = -\frac{1}{4\pi^2} \sum_{n=0}^{\infty} \varepsilon_n \cos n(\varphi - \varphi_1) \int_{-\infty}^{+\infty} \frac{H_n(k_r a)}{k_r a H_n(k_r a)} e^{jk_z | z - z_1 |} dk_z,$$
(6)

where $k^2 = k_z^2 + k_r^2$, $H_n()$ is the *n*th-order Hankel function, and $H'_n()$ denotes the derivative of the Hankel function with respect to its arguments; ε_n is Neumann factor.

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3. THE SHELL RESPONSE

As in most studies, the modal analysis method is addressed again to obtain the response of the shell. The response of the simply supported shell subjected to the moving load is a combination of harmonic motions at discrete frequencies $N\Omega$ (cf. reference [13]). The shell displacement vector in equation (1) can be easily expanded as a series of the *in vacuo* simply supported shell modes

$$U = \sum_{\alpha=0}^{1} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm}^{\alpha} \Phi_{nm}^{\alpha},$$
(7)

where Φ_{nm}^{α} and A_{nm}^{α} are the eigenvector and the modal amplitude of the (α, n, m) shell modes respectively. Φ_{nm}^{α} has its form as follows:

$$\Phi_{nm}^{\alpha} = \begin{pmatrix} D_{nm} \sin\left(n\varphi + \alpha \frac{\pi}{2}\right) \cos\frac{m\pi z}{L} \\ E_{nm} \cos\left(n\varphi + \alpha \frac{\pi}{2}\right) \sin\frac{m\pi z}{L} \\ \sin\left(n\varphi + \alpha \frac{\pi}{2}\right) \sin\frac{m\pi z}{L} \end{pmatrix}.$$
(8)

where $(D_{nm}, E_{nm}, 1)$ are components of the eigenvector, $\alpha = 0$ (resp. 1) denotes anti-symmetric (resp. symmetric) shell modes, and *n*, *m* respectively the circumferential order and the longitudinal order.

Inserting equation (7) into equation (1), and using the orthogonality property of the shell modes, equation (1) can be transformed as

$$M_{nm} [\omega_{nm}^{2} (1 - j\eta) - \omega^{2}] A_{nm}^{\alpha} = f_{nm}^{\alpha} - p_{nm}^{\alpha}, \qquad (9)$$

where K_{nm} denotes the generalized stiffness of the shell mode, M_{nm} its generalized mass, ω_{nm} its natural angular frequencies. f_{nm}^{α} is modal exciting force, which is given in reference [13] as follows:

$$f_{nm}^{\alpha} = \frac{F_0}{2} \sin\left(\frac{m\pi z_0}{L}\right) \tau_{Nn}^{\alpha},\tag{10}$$

with

$$\tau_{Nn}^{\alpha} = \begin{cases} 0, & N \neq n, \\ 0, & N = n = \alpha = 0, \\ 2, & N = n = 0, \alpha = 1, \\ j, & N = n \neq 0, \alpha = 0, \\ 1, & N = n \neq 0, \alpha = 1. \end{cases}$$
(11)

Finally, p_{nm}^{α} denotes modal sound pressure widely known as

$$p_{nm}^{\alpha} = -j\omega \sum_{q=1}^{\infty} Z_{nmq} A_{nq}^{\alpha}, \qquad (12)$$

where Z_{nmq} is the radiation impedance of the shell (cf. reference [8]). It expresses the modal coupling between the longitudinal shell modes (*m* and *q*) due to the fluids. Inserting equation (12) into equation (9), equation (9) can be rewritten as

inserting equation (12) into equation (9), equation (9) can be rewritten as

$$\left\{M_{nm}\left[\omega_{nm}^{2}(1-j\eta)-\omega^{2}\right]-j\omega Z_{nmm}\right\}A_{nm}^{\alpha}=f_{nm}^{\alpha}+j\omega\sum_{\substack{q=1\\q\neq m}}^{\infty}Z_{nmq}A_{nq}^{\alpha},\qquad(13)$$

where η is the loss factor of the shell material, Z_{nmm} is the radiation self-impedance, and $Z_{nmq} (q \neq m)$ is the radiation interaction impedance.

Laulagnet and Guyader [1] point out that while the interaction impedance $Z_{nmq}(q \neq m)$ has a non-negligible influence on sound radiated power, it has very weak influence on vibration velocity. However, when $Z_{nmq}(q \neq m)$ is neglected, even in radiated power, the general characteristics of the frequency spectrum curves remain unchanged, though an overestimation of the radiated power is generally obtained. Because the sound radiation ability (the radiated power and the mean quadratic velocity) is mainly studied in this paper, for a qualitative analysis, the interaction impedance $Z_{nmq}(q \neq m)$ can be neglected and an approximation considered sufficient.

According to the above approximation, equation (13) can be simplified as

$$A_{nm}^{\alpha} = \frac{f_{nm}^{\alpha}}{\left[M_{nm}(\omega_{nm}^2 - \omega^2) - \omega X_{nmm}\right] - j(\eta M_{nm}\omega_{nm}^2 + \omega R_{nnm})},$$
(14)

where R_{nmm} is the radiation self-resistance (real part of Z_{nmm}), and X_{nmm} is the radiation self-reactance (imaginary part of Z_{nmm}). X_{nmm} introduces the added mass and R_{nmm} the modal loss by radiation.

Equation (11) shows that the shell can be excited at the frequency $N\Omega$ only when the harmonic order N of the rotating exciting force is equal to the circumferential mode order n of the shell. This contrasts with the case of a stationary harmonic point force, where all circumferential modes can be excited by the force. Hence, by substituting ω in equation (14) with $N\Omega$, the modal amplitude of the shell can be obtained:

$$A_{Nm}^{\alpha} = \frac{(F_0/2)\sin(m\pi z_0/L)\tau_{N\alpha}}{[M_{Nm}(\omega_{Nm}^2 - (N\Omega)^2) - N\Omega X_{Nmm}] - j(\eta M_{Nm}\omega_{Nm}^2 + N\Omega R_{Nmm})}.$$
 (15)

Accordingly, one gets the following expression:

$$|A_{Nm}^{\alpha}| = \frac{(F_0/2)\sin(m\pi z_0/L)\tau_{N\alpha}}{M_{Nm}\omega_{Nm}^2} \cdot \frac{1}{\sqrt{(1-\hat{X}\lambda-\lambda^2)^2+(\eta+\hat{R}\lambda)^2}},$$
 (16)

with

$$\lambda = \frac{N\Omega}{\omega_{Nm}}, \qquad \hat{X} = \frac{X_{Nmm}}{M_{Nm}\omega_{Nm}}, \qquad \hat{R} = \frac{R_{Nmm}}{M_{Nm}\omega_{Nm}}.$$
(17)

Equation (16) shows that $|A_{Nm}^{\alpha}|$ becomes maximum at resonance only when

$$2\lambda^3 + 3\hat{X}\lambda^2 + (\hat{X}^2 - 2 + \hat{R}^2)\lambda + \eta\hat{R} - \hat{X} = 0.$$
(18)

Equation (18) can be easily solved and a rational root signed as λ_0 is obtained. Thus, the vibration critical speed is given by

$$\Omega_c = \lambda_0 \omega_{Nm} / N = \bar{\omega}_{Nm} / N, \tag{19}$$

where $\bar{\omega}_{Nm}$ denotes the resonant angular frequency of the shell. In particular, it is easily seen that λ_0 equals unity when the shell is *in vacuo*, that is to say, in this case $\bar{\omega}_{Nm} = \omega_{Nm}$. Furthermore, of practical importance is the first vibration critical speed $\Omega_c^{(v)}$ of the rotating load, defined with respect to N and m as

$$\Omega_c^{(\nu)} = \min_{N,m}(\bar{\omega}_{Nm}/N). \tag{20}$$

Equation (20) indicates that the first vibration critical speed of the shell in fluid is lower than the one *in vacuo*, because of the radiation self-impedance.

4. SOUND RADIATION ANALYSIS OF THE SHELL

In this paper, we focus our attention on global quantities to study shell acoustic radiation. More precisely, we are interested in the calculation of vibroacoustic indicators consisting of three main coefficients: the mean radial quadratic velocity of the shell $\langle \bar{u}_r^2 \rangle$, the radiated sound power W_{rad} in the fluid, and the radiation efficiency σ_{rad} . These indicators are global in nature, and they are calculated at the harmonic angular frequencies $N\Omega$ of the load rotational speed Ω . It can be shown that they are given by (the radiation interaction impedance Z_{nmq} is neglected)

$$\langle \bar{u}_{r}^{2} \rangle = \frac{(N\Omega)^{2}}{4} \sum_{\alpha=0}^{1} \sum_{m=1}^{m_{max}} |A_{Nm}^{\alpha}|^{2}, \quad W_{rad} = \frac{(N\Omega)^{2}}{2} \sum_{\alpha=0}^{1} \sum_{m=1}^{m_{max}} R_{Nmm} |A_{Nm}^{\alpha}|^{2},$$
$$\sigma_{rad} = \frac{W_{rad}}{\rho_{0} c_{0} S \langle \bar{u}_{r}^{2} \rangle}.$$
(21)

Here the series of the longitudinal mode *m* are truncated to m_{max} . The selection of the values of m_{max} is discussed in detail in reference [1].

In addition, it is well known that, when the shell circumferential wavelength equals the acoustic wavelength in the exterior medium, an intensive sound radiation will happen. Hence, it is necessary to define the radiation critical rotating speed as follows:

$$\Omega_c^{(a)} = c_0/a. \tag{22}$$

In this equation, the radiation critical rotating speed depends on the radius of the shell *a* and the sound speed c_0 in the exterior fluid. Equation (22) indicates that a fluid-loaded cylindrical shell excited by a rotating load will have strong sound radiation efficiency if the load travels at a speed greater than or equal to $\Omega_c^{(a)}$.

5. NUMERICAL RESULTS AND DISCUSSION

In this section, the numerical studies are based on a simply supported steel shell of which the geometrical and mechanical parameters are as follows: length L = 2.0 m, radius a = 0.3 m thickness h = 0.005 m, density $\rho = 7850$ kg/m³, Young's modulus E = 211 GPa, Poisson ratio v = 0.3, structural loss factor $\eta = 0.01$. The shell is supposed to be immersed in air and water, and water is characterized by density $\rho_0 = 1000$ kg/m³ and sound speed $c_0 = 1500$ m/s. The shell is excited at $z_0 = 1.0$ m and the magnitude of the rotating force is $F_0 = 100$ N.

According to equation (20), whether this particular shell is *in vacuo* or in air, one can easily get the first vibration critical speeds (in cycles per second). They are the minimum values of $(1/2\pi)(\bar{\omega}_{Nm}/N)$ for all possible values of N and m. In this case, they both correspond to N = 3 and m = 1, and their values are $(1/2\pi)\Omega_c^{(\nu)} = 45.5$ Hz (see Figure 2). Hence, for such a rotating speed, the (3, 1) shell mode *in vacuo* or in air is resonant. In addition, the first vibration critical speed in water is not given here because it depends on the rotating speed Ω . In addition, according to equation (22), the radiation critical speeds $(1/2\pi)\Omega_c^{(a)}$ for this particular shell are 182 Hz in air and 796 Hz in water. Thus, in the following numerical studies, five rotating speeds (in cycles per second) $(\Omega/2\pi)$, i.e. 30, 120, 240, 400 and 800 Hz are selected.

Figures 3-6 show the influences of air and water on the resonant angular frequency $\bar{\omega}_{Nm}$ versus the circumferential order N of the shell, for various values of the longitudinal order m and the rotating speed $(\Omega/2\pi)$. The resonant angular frequencies in air are equal to those *in vacuo* at any value of the rotating speed $(\Omega/2\pi)$; that is to say, air has no influence on the natural angular frequencies of the shell *in vacuo* modes. However, when the fluid is water, the natural angular frequencies of the shell *in vacuo* modes decrease obviously as the rotating speed increases, because of a strong coupling between the shell and water.



Figure 2. Vibration critical speeds versus N of the shell in vacuo or in air.



Figure 3. The influence of air on $\overline{\omega}_{Nm}$ versus N for selected $\Omega/2\pi(m = 1)$. Key: —, in vacuo; -o-, $\Omega/2\pi = 30$ Hz; - \bullet -, $\Omega/2\pi = 120$ Hz; - \triangle -, $\Omega/2\pi = 240$ Hz; - \bullet -, $\Omega/2\pi = 400$ Hz;- \Box -, $\Omega/2\pi = 800$ Hz.



Figure 4. The influence of air on $\bar{\omega}_{Nm}$ versus N for selected $\Omega/2\pi$ (m = 4). Key: —, in vacuo; -o-, $\Omega/2\pi = 30$ Hz; - \bullet -, $\Omega/2\pi = 120$ Hz; - Δ -, $\Omega/2\pi = 240$ Hz; - \bullet -, $\Omega/2\pi = 400$ Hz;- \Box -, $\Omega/2\pi = 800$ Hz.

Figures 7–11 show the mean radial quadratic velocities of the shell in air and water for the rotating speeds considered above as a function of the frequency. These spectra are made of discrete harmonics of the rotating speed due to the nature of the generalized force (see equations (10) and (11)). Both in air and water, it can be seen that for the rotational speed ($\Omega/2\pi$) (30 Hz) less than ($1/2\pi$) $\Omega_c^{(v)}$ (45.5 Hz), there is only one maximum. For the case of rotating speeds (120, 240 400 and 800 Hz) greater than ($1/2\pi$) $\Omega_c^{(v)}$, more and more shell modes are excited. In addition, the mean radial quadratic velocities decrease as the rotating speeds increase.

Figures 12–16 and 17–21 show respectively the radiated sound powers and the radiation efficiencies of the shell in air and water for the five rotating speeds selected above. It is noted that when the rotating speeds (60 and 120 Hz) are lower than the radiation critical speed $(1/2\pi)\Omega_c^{(a)}$ of the shell in air (182 Hz), whether in air or in



Figure 5. The influence of water on $\bar{\omega}_{Nm}$ versus N for selected $\Omega/2\pi (m = 1)$. Key: —, in vacuo; $-\circ -$, $\Omega/2\pi = 30$ Hz; $-\bullet -$, $\Omega/2\pi = 120$ Hz; $-\Delta -$, $\Omega/2\pi = 240$ Hz; $-\bullet -$, $\Omega/2\pi = 400$ Hz; $-\Box -$, $\Omega/2\pi = 800$ Hz.



Figure 6. The influence of water on $\bar{\omega}_{Nm}$ versus N for selected $\Omega/2\pi$ (m = 4). Key: —, in vacuo; -o-, $\Omega/2\pi = 30$ Hz; - \bullet -, $\Omega/2\pi = 120$ Hz; - Δ -, $\Omega/2\pi = 240$ Hz; - \bullet -, $\Omega/2\pi = 400$ Hz;- \Box -, $\Omega/2\pi = 800$ Hz.



Figure 7. Mean radial quadratic velocity of the shell in different fluids ($\Omega/2\pi$ 30 Hz). Key: -•-, air; -o-, water.



Figure 8. As for Figure 7, but $\Omega/2\pi = 120$ Hz.



Figure 9. As for Figure 7, but $\Omega/2\pi = 240$ Hz.



Figure 10. As for Figure 7, but $\Omega/2\pi = 400$ Hz.



Figure 11. As for Figure 7, but $\Omega/2\pi = 800$ Hz.



Figure 12. Radiated sound power of the shell in different fluids ($\Omega/2\pi = 30$ Hz). Key: $-\Phi$, air; $-\Theta$, water.



Figure 13. As for Figure 12, but $\Omega/2\pi = 120$ Hz.



Figure 14. As for Figure 12, but $\Omega/2\pi = 240$ Hz.



Figure 15. As for Figure 12, but $\Omega/2\pi = 400$ Hz.



Figure 16. As for Figure 12, but $\Omega/2\pi = 800$ Hz.



Figure 17. Radiation efficiency of the shell in different fluids ($\Omega/2\pi = 30$ Hz). Key: - \bullet -, air; - \circ -, water.



Figure 18. As for Figure 17, but $\Omega/2\pi = 120$ Hz.



Figure 19. As for Figure 17, but $\Omega/2\pi = 240$ Hz.



Figure 20. As for Figure 17, but $\Omega/2\pi = 400$ Hz.



Figure 21. As for Figure 17, but $\Omega/2\pi = 800$ Hz.

water, the radiated sound powers are small and the radiation efficiencies are weak. For rotating speeds (240 and 400 Hz) greater than 182 Hz, both the radiated sound powers and the radiation efficiencies in air are strong. However, in this case, the radiation of the shell in water remains weak. When the rotating speed (800 Hz) is greater than the radiation critical speed $(1/2\pi)\Omega_c^{(a)}$ of the shell in water (796 Hz), a strong radiation is obtained in water or in air, and the corresponding values of the radiated powers and radiation efficiencies in water are bigger than those in air. In this case, an interesting phenomenon appears in which for air strong radiation efficiency gives strong power radiation. This conclusion is in correspondence with the one given in reference [1], in which a non-moving load was considered.

6. CONCLUSIONS

In this paper, a theoretical model has been developed to analyze the vibroacoustic behaviour of a simply supported cylindrical shell immersed in fluids,

subjected to a constant point load travelling continuously along its circumference. It has been shown that the frequency component is a series of equal amplitude harmonics $N\Omega$ of the rotating speed Ω and that each of these harmonics excites exclusively the circumferential mode n = N. As a result, the shell will only exhibit vibration and sound radiation at these harmonic frequencies. This contrast with the case of a stationary harmonic point load where all circumferential modes are excited at the frequency of excitation.

In addition, it has been shown that the vibroacoustic behaviour of a shell immersed in a heavy fluid is completely different from that of one immersed in a light fluid. In particular, when the rotational speed is greater or equal to the radiation critical speed $\Omega_c^{(a)}$, an important conclusion is obtained as the same case of non-moving excitation (cf. reference [1]). It is shown that for light fluids strong radiation efficiency is equivalent to strong power radiation. However, this is not the case for heavy fluids for which an inverse situation occurs: strong power radiation is obtained for low radiation efficiency, and weak power radiation for high radiation efficiency.

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